One formula for r is:

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}$$

Dividing numerator and denominator by N:

$$r = \frac{\sum xy \big/ N}{\sqrt{\sum x^2 \sum y^2 \big/ N}}$$

Note that x and y are assumed to be in deviation-score form (means of 0).

Letting the two parallel forms of the test be x_1 and x_2

$$r = \frac{\sum x_1 x_2 \big/ N}{\sqrt{\sum x_1^2 \sum x_2^2 \big/ N}}$$

Separating x_1 and x_2 into true and error components, the numerator becomes:

$$\frac{\sum (t + e_1)(t + e_2)}{N}$$

Note that the two forms have the same true score (for a given observation) but different errors of measurement.

Expanding terms, the numerator becomes:

$$\frac{\sum t^2 + \sum te_1 + \sum te_2 + \sum e_1 e_2}{N}$$

Since the true scores and the errors of measurement are uncorrelated and each has a mean of 0, the terms that include error vanish since the average cross-product is 0.

$$\frac{\sum t^2}{N}$$

which is the variance of true scores:

$$\sigma^2_{True}$$

The denominator is the square root of the product of the two test variances. The two variances are:

$$\frac{\sum x_1^2}{N}\ and\ \frac{\sum x_2^2}{N}$$

The square root of the product of the variances is simply the variance. The variances of x_1 and x_2 are equal and are denoted by as:

$$\sigma^2_{Test}$$

$$\sqrt{\sigma^2_{Test}\ \sigma^2_{Test}} = \sigma^2_{Test}$$

Therefore, the correlation between the two parallel forms is:

$$\frac{\sigma^2_{True}}{\sigma^2_{Test}}$$